ELSEVIER

Available at
www.ElsevierMathematics.com
POWERED BY SCIENCE @ DIRECT®

Applied Mathematics Letters 16 (2003) 1283–1287

Applied Mathematics Letters

www.elsevier.com/locate/aml

Infinitely Many Homoclinic Orbits for the Second-Order Hamiltonian Systems

WENMING ZOU*
Department of Mathematical Sciences, Tsinghua University
Beijing 100084, P.R. China

SHUJIE LI†
Academy of Mathematics and Systems Sciences
Academia Sinica, Beijing 100080, P.R. China

(Received and accepted October 2002)

Communicated by G. Wake

Abstract—In this paper, the existence of homoclinic orbits for the second-order Hamiltonian systems without periodicity is studied and infinitely many homoclinic orbits for both superlinear and asymptotically linear cases are obtained.

Keywords—Homoclinic orbits, New fountain theorem, Hamiltonian systems.

1. INTRODUCTION

In this article, we are concerned with the existence of homoclinic orbits of the second-order system

$$-\ddot{z} + L(t)z = H_z(t, z), \tag{HS}$$

where $z = (z_1, z_2, \dots, z_N)$, $L(t) \in \mathcal{C}(\mathbf{R}, \mathbf{R}^{N^2})$ is a symmetric matrix valued function, $H \in \mathcal{C}^1(\mathbf{R} \times \mathbf{R}^N, \mathbf{R})$, $H_z(t, z) = \nabla_z H(t, z)$. A nontrivial solution z of (HS) is said to be homoclinic to zero if $z \in \mathcal{C}^2(\mathbf{R}, \mathbf{R}^N)$, $z(t) \to 0$ and $\dot{z}(t) \to 0$ as $|t| \to \infty$.

1.1. Infinitely Many Homoclinic Orbits for Superlinear Case

We make the following assumptions. From now on, we denote by the letter c the various positive constants where the exact values are irrelevant.

(a_1) There exists $\mu > 2$ such that $c|z|^\mu \le H_z(t, z) \cdot z$, $\forall\,(t, z) \in \mathbf{R} \times \mathbf{R}^N$.

(a_2) For any $\varepsilon > 0$, there exists $C_\varepsilon > 0$ such that $|H_z(t, z)| \le \varepsilon|z| + C_\varepsilon|z|^{\mu-1}$, $\forall\,(t, z) \in \mathbf{R} \times \mathbf{R}^N$.

(a_3) There are $\mu_0 > 2$, $R_0 > 0$, $\alpha > \mu(\mu - 2)/(\mu - 1)$ such that $H_z(t, z) \cdot z \ge \mu_0 H(t, z)$ for $|z| \le R_0$, $t \in \mathbf{R}$; $H_z(t, z) \cdot z - 2H(t, z) \ge c|z|^\alpha$ for $|z| \ge R_0$, $t \in \mathbf{R}$ uniformly for $(t, z) \in \mathbf{R} \times \mathbf{R}^N$.

---

*The author to whom all correspondence should be addressed. Present address: Department of Mathematics, University of California, Irvine, CA 92697-3875, U.S.A. E-mail: wzou@math.uci.edu. Supported by NSFC (No. 10001019) and AMSS, Academia Sinica.
†Supported by NSFC (973) and NSF-AS.

0893-9659/03/$ - see front matter Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX
doi: 10.1016/S0893-9659(03)00191-5

THEOREM 1.1. *Assume that* (a_1)–(a_3) *hold and that* H *is even in* z. *Furthermore,*

$$l(t) := \inf_{|z|=1} L(t)z \cdot z \to \infty, \qquad \text{as } |t| \to \infty. \tag{1.1}$$

Then (HS) has infinitely many homoclinic orbits.

REMARK 1.1. There are many papers studying the existence of (multiple) homoclinic orbits for superlinear cases (see, e.g., [1–8]). Particularly, [2,6–8] considered periodic or almost periodic potential cases. We emphasize that the results in all the papers mentioned above were obtained under the global Ambrosetti-Rabinowitz type condition

$$0 < \mu H(t,z) \le H_z(t,z) \cdot z, \quad \forall (t,z) \in \mathbf{R} \times \mathbf{R}^N; \qquad \mu > 2 \text{ is a constant.} \tag{1.2}$$

In this paper, we do not need the periodicity assumption and consider other kinds of conditions. We can easily give an example which satisfies (a_1)–(a_3) but not (1.2).

1.2. Asymptotically Linear Cases

We consider the following conditions.

(b_1) $H(t,z) = (1/2)\beta|z|^2 + G(t,z)$, where $\beta \notin \sigma(-\frac{d^2}{dt^2} + L(t))$; σ denotes the spectrum.

(b_2) There exists $\delta_i \in (1,2)$, $i = 1,2$, such that $c|z|^{\delta_1} \le G(t,z)$, $G(t,0) \equiv 0$, $|G_z(t,z)| \le c|z|^{\delta_2 - 1}$, for all $(t,z) \in \mathbf{R} \times \mathbf{R}^N$.

THEOREM 1.2. *Assume that* (b_1) *and* (b_2) *hold and that* H *is even in* z. *Furthermore,*

$$l(t)|t|^{p-2} \to \infty, \qquad \text{as } |t| \to \infty, \quad \text{where constant } p < 1. \tag{1.3}$$

Then (HS) has infinitely many homoclinic orbits.

2. PROOF OF THEOREM 1.1

We denote by A the self-adjoint extension of the operator $-(\frac{d^2}{dt^2} + L(t))$ with the domain $\mathcal{D} \subset L^2 := L^2(\mathbf{R}, \mathbf{R}^N)$. Let $E := \mathcal{D}(|A|^{1/2})$, the domain of $|A|^{1/2}$, and define on E the inner product and norm $(z,w)_0 := (|A|^{1/2}z, |A|^{1/2}w)_2 + (z,w)_2$, $\|z\|_0 := (z,z)_0^{1/2}$, where $(\cdot,\cdot)_2$ denotes the inner product of L^2. Then E is a Hilbert space. Furthermore, if (1.1) holds, then E is compactly embedded in $L^p := L^p(\mathbf{R}, \mathbf{R}^N)$ for $2 \le p \le \infty$ (see, e.g., [4, Lemma 2.1]). Therefore, the spectrum $\sigma(A)$ consists of eigenvalues numbered by $\lambda_1 \le \lambda_2 \le \lambda_n \cdots \nearrow \infty$ (counted in their multiplicities) and a corresponding system of eigenfunctions $\{e_n\}$, $Ae_n = \lambda_n e_n$ which forms the orthonormal basis of L^2. Assume $\lambda_1, \dots, \lambda_{n^-} < 0$, $\lambda_{n^-+1} = \cdots = \lambda_{n^*} = 0$ and let $E^- := \operatorname{span}\{e_1, \dots, e_{n^-}\}$, $E^0 := \operatorname{span}\{e_{n^-+1}, \dots, e_{n^*}\}$, $E^+ := \overline{\operatorname{span}\{e_{n^*+1}, \dots\}}$. Then $E = E^- \oplus E^0 \oplus E^+$. We introduce on E the following inner product $(z,w) := (|A|^{1/2}z, |A|^{1/2}w)_2 + (z^0, w^0)_2$, $\|z\|^2 := (z,z)$, where $z = z^- + z^0 + z^+$ and $w = w^- + w^0 + w^+ \in E = E^- \oplus E^0 \oplus E^+$. Then $\|\cdot\|$ and $\|\cdot\|_0$ are equivalent. Set

$$\Phi(z) := \frac{1}{2}\int_{\mathbf{R}} \left(|\dot z|^2 + L(t)z \cdot z\right) dt - \int_{\mathbf{R}} W(t,z)\, dt = \frac{1}{2}\left\|z^+\right\|^2 - \frac{1}{2}\left\|z^-\right\|^2 - \int_{\mathbf{R}} W(t,z)\, dt.$$

Then $\Phi \in C^1(E, \mathbf{R})$ and the critical points of Φ correspond to the homoclinic orbits of (HS). To prove Theorem 1.1, we shall use the following fountain theorem obtained in [9]. Let E be a Banach space with the norm $\|\cdot\|$ and $E = \overline{\oplus_{j \in \mathbf{N}} X_j}$ with $\dim X_j < \infty$ for any $j \in \mathbf{N}$. Set $Y_k = \oplus_{j=0}^k X_j$, $Z_k = \overline{\oplus_{j=k}^\infty X_j}$ and $B_k = \{u \in Y_k : \|u\| \le \rho_k\}$, $N_k = \{u \in Z_k : \|u\| = r_k\}$ for $\rho_k > r_k > 0$. Consider C^1-functional $\Phi_\lambda : E \to \mathbf{R}$ defined by $\Phi_\lambda(z) := A(z) - \lambda B(z)$, $\lambda \in [1,2]$. Assume that

(F_1) Φ_λ maps bounded sets to bounded sets uniformly for $\lambda \in [1,2]$. Furthermore, $\Phi_\lambda(-z) = \Phi_\lambda(z)$ for all $(\lambda, z) \in [1,2] \times E$.

(F_2) $B(z) \ge 0$ for all $z \in E$; $A(z) \to \infty$ or $B(z) \to \infty$ as $\|z\| \to \infty$; or

(F_2') $B(z) \le 0$ for all $z \in E$; $B(z) \to -\infty$ as $\|z\| \to \infty$.

Let, for $k \geq 2$, $\Gamma_k := \{\gamma \in \mathcal{C}(B_k, E) : \gamma \text{ is odd}, \gamma|_{\partial B_k} = id\}$, $c_k(\lambda) := \inf_{\gamma\in\Gamma_k} \max_{z\in B_k} \Phi_\lambda(\gamma(z))$, $b_k(\lambda) := \inf_{z\in Z_k,\, \|z\|=r_k} \Phi_\lambda(z)$, $a_k(\lambda) := \max_{z\in Y_k,\, \|z\|=\rho_k} \Phi_\lambda(z)$. Now we have the following theorem (cf. [9, Theorem 2.1, p. 345]).

THEOREM 2.1. *Assume (F_1) and (F_2) (or (F_2')). If $b_k(\lambda) > a_k(\lambda)$ for all $\lambda \in [1,2]$, then $c_k(\lambda) \geq b_k(\lambda)$ for all $\lambda \in [1,2]$. Moreover, for a.e. $\lambda \in [1,2]$, there exists a sequence $\{z_n^k(\lambda)\}_{n=1}^\infty$ such that $\sup_n \|z_n^k(\lambda)\| < \infty$, $\Phi_\lambda'(z_n^k(\lambda)) \to 0$ and $\Phi_\lambda(z_n^k(\lambda)) \to c_k(\lambda)$ as $n \to \infty$.*

Define $\Phi_\lambda(z) := (1/2)\|z^+\|^2 - \lambda((1/2)\|z^-\|^2 + \int_{\mathbf{R}} H(t,z)\,dt) := A(z) - \lambda B(z)$ and $X_j := \mathbf{R}e_j$. We first have the following lemmas. We denote by $|\cdot|_s$ the norm of L^s.

LEMMA 2.1. *There exists ρ_k large enough such that $a_k(\lambda) := \max_{z\in Y_k, \|z\|=\rho_k} \Phi_\lambda(z) \leq 0$ for all $\lambda \in [1,2]$.*

PROOF. By Condition (a_1), $H(t,z) \geq c|z|^\mu$ for all (t,z). Let $z \in Y_k$, $z = z^- + z^0 + z^+$, and noting that $\dim Y_k < \infty$, it is easily seen that $\Phi_\lambda(z) \leq 0$ for $\|z\| := \rho_k$ large enough. ∎

LEMMA 2.2. *There exist $r_k > 0$, $\bar{b}_k \to \infty$ such that $b_k(\lambda) := \inf_{z\in Z_k,\, \|z\|=r_k} H_\lambda(z) \geq \bar{b}_k$, for all $\lambda \in [1,2]$.*

PROOF. Set $\beta_k := \sup_{z\in Z_k,\, \|z\|=1} |z|_\mu$. Then $\beta_k \to 0$ since E is imbedded compactly into L^μ. Choose k large enough such that $Z_k \subset E^+$. By Condition (a_2), we see that, for ε small enough, there exists C_ε such that $H(t,z) \leq \varepsilon|z|^2 + C_\varepsilon|z|^\mu$ for all (t,z). Hence,

$$\Phi_\lambda(z) \geq \frac{1}{2}\|z\|^2 - \lambda\varepsilon|z|_2^2 - \lambda C_\varepsilon|z|_\mu^\mu \geq \frac{1}{4}\|z\|^2 - c\beta_k^\mu\|z\|^\mu.$$

Let $r_k := (8c\beta_k)^{1/(2-\mu)}$. Then for $z \in Z_k$ with $\|z\| = r_k$, we have that $\Phi_\lambda(z) \geq c(\beta_k^\mu)^{2/(2-\mu)} := \bar{b}_k \to \infty$, uniformly for λ as $k \to \infty$. ∎

By Theorem 2.1, we have the following.

LEMMA 2.3. *There exist $\lambda_n \to \infty$ as $n \to \infty$, $\{z_n(k)\}_{n=1}^\infty \subset E$ such that $\Phi_{\lambda_n}'(z_n(k)) = 0$, $\Phi_{\lambda_n}(z_n(k)) \in [\bar{b}_k, \bar{c}_k]$, where $\bar{c}_k := \sup_{z\in B_k} \Phi(z)$.* ∎

PROOF OF THEOREM 1.1. By Lemma 2.3, it suffices to prove the boundedness of $\{z_n(k)\}_{n=1}^\infty$. By Condition (a_1) and Lemma 2.3, we have that $\|z_n(k)^+\|^2 - \lambda_n\|z_n(k)^-\|^2 \geq c|z_n(k)|_\mu^\mu$ and $(1/2 - 1/\mu_0)(\|z_n(k)^+\|^2 - \lambda_n\|z_n(k)^-\|^2) + \lambda_n \int_{\mathbf{R}}((1/\mu_0)H_z(t,z_n(k)) \cdot z_n(k) - H(t,z_n(k))) \leq \bar{c}_k$. Therefore, by ($a_2$),

$$\begin{aligned}\left\|z_n(k)^+\right\|^2 - \lambda_n\left\|z_n(k)^-\right\|^2 &\leq c + c\left(\frac{1}{2} - \frac{1}{\mu_0}\right)\int_{|z_n(k)|\geq R_0} H_z(t,z_n(k)) \cdot z_n(k)\,dt \\ &\leq c + c\int_{|z_n(k)|\geq R_0} \left(|z_n(k)|^\mu + |z_n(k)|^2\right)\,dt \\ &\leq c + c\int_{|z_n(k)|\geq R_0} |z_n(k)|^\mu\,dt,\end{aligned}$$

by (2.1), $|z_n(k)|_\mu^\mu \leq c + c\int_{|z_n(k)|\geq R_0} |z_n(k)|^\mu\,dt$. By (2.2), Lemma 2.3, and (a_3), we have $c \geq \int_{\mathbf{R}}((1/2)H_z(t,z_n(k))\cdot z_n(k) - H(t,z_n(k)))\,dt \geq c\int_{|z_n(k)|\geq R_0} |z_n(k)|^\alpha\,dt$. Without loss of generality, we assume that $\alpha < \mu$. Choosing $t \in ((\mu-\alpha)/\mu, 1)$, we assume that $t < 2(\mu-\alpha)/\mu(2-\alpha)$. If $\alpha < 2$, then $s := t\mu\alpha/(\alpha - (1-t)\mu) \geq 2$, and therefore,

$$\int_{|z_n|\geq R_0} |z_n|^\mu \leq \left(\int_{|z_n|\geq R_0} |z_n|^\alpha\right)^{(1-t)\mu/\alpha} \left(\int_{|z_n|\geq R_0} |z_n|^s\right)^{t\mu/s} \leq c\|z_n\|^{t\mu},$$

therefore, $|z_n(k)|_\mu^\mu \leq c + c\|z_n\|^{t\mu}$. On the other hand, by Condition (a_2) and Lemma 2.3,

$$\begin{aligned}\left\|z_n(k)^+\right\|^2 &= \lambda_n \int_{\mathbf{R}} H_z(t,z_n(k)) \cdot z_n(k)^+\,dt \\ &\leq c\int_{\mathbf{R}} \left(\varepsilon|z_n(k)| + C_\varepsilon|z_n(k)|^{\mu-1}\right)\left|z_n(k)^+\right|\,dt \\ &\leq \varepsilon c\|z_n(k)\|^2 + c + cC_\varepsilon\|z_n\|^{t(\mu-1)+1}.\end{aligned}$$

Similarly, $\|z_n(k)^-\|^2 \le \varepsilon c\|z_n(k)\|^2 + c + cC_\varepsilon\|z_n\|^{t(\mu-1)+1}$. If $E^0 = \{0\}$, then $\{z_n(k)\}_{n=1}^\infty$ is bounded. If $E^0 \ne \{0\}$, then $|z_n(k)^0|_2^2 = (z_n(k)^0, z_n(k))_2 \le |z_n(k)^0|_{\mu'}|z_n(k)|_\mu$. Noting that $\dim E^0 < \infty$, we observe that

$$\left\|z_n(k)^0\right\|^2 \le c|z_n(k)|_\mu^2 \le c\|z_n(k)\|^{2t} + c \le c + c\left\|z_n(k)^0\right\|^{2t} + c\left\|z_n(k)^+\right\|^{2t} + c\left\|z_n(k)^-\right\|^{2t}.$$

Combining the above arguments, $\{z_n(k)\}_{n=1}^\infty$ is bounded. Since E is embedded compactly into L^p $(p \ge 2)$, then by standard argument, we see that $\{z_n(k)\}_{n=1}^\infty$ has a strong convergent subsequence, and hence, Φ has a critical point w_k with $\Phi(w_k) \in [\bar{b}_k, \bar{c}_k]$. Consequently, we get infinitely many solutions since $\bar{b}_k \to \infty$. ∎

3. PROOF OF THEOREM 1.2

First, we recall the following abstract result (cf. [9, Theorem 2.2, p. 349]) which will be applied in proving Theorem 1.2.

THEOREM 3.1. *The C^1-functional $\Phi_\lambda : E \to \mathbf{R}$ defined by $\Phi_\lambda(z) = A(z) - \lambda B(z)$, $\lambda \in [1,2]$, satisfies the following.*

- (T_1) *Φ_λ maps bounded sets to bounded sets uniformly for $\lambda \in [1,2]$. Moreover, $\Phi_\lambda(-z) = \Phi_\lambda(z)$ for all $(\lambda, z) \in [1,2] \times E$.*
- (T_2) *$B(z) \ge 0$; $B(z) \to \infty$ as $\|z\| \to \infty$ on any finite-dimensional subspace of E.*
- (T_3) *There exist $\rho_k > r_k > 0$ such that $a_k(\lambda) := \inf_{z\in Z_k,\, \|z\|=\rho_k} \Phi_\lambda(z) \ge 0 > b_k(\lambda) := \max_{z\in Y_k,\, \|z\|=r_k} \Phi_\lambda(z)$ for all $\lambda \in [1,2]$ and $d_k(\lambda) := \inf_{z\in Z_k,\, \|z\|\le\rho_k} \Phi_\lambda(z) \to 0$ as $k \to \infty$ uniformly for $\lambda \in [1,2]$.*

Then there exist $\lambda_n \to 1$, $z(\lambda_n) \in Y_n$ such that $\Phi'_{\lambda_n}|_{Y_n}(z(\lambda_n)) = 0$, $\Phi_{\lambda_n}(z(\lambda_n)) \to c_k \in [d_k(2), b_k(1)]$ as $n \to \infty$. Particularly, if $\{z(\lambda_n)\}$ has a convergent subsequence for every k, then Φ_1 has infinitely many nontrivial critical points $\{z_k\} \in E \setminus \{0\}$ satisfying $\Phi_1(z_k) \to 0^-$ as $k \to \infty$.

LEMMA 3.1. *Under the assumptions of Theorem 1.2, there exists r_k small enough such that $b_k(\lambda) := \max_{z\in Y_k,\, \|z\|=r_k} \Phi_\lambda(z) < 0$ uniformly for $\lambda \in [1,2]$.*

PROOF. Let $u \in Y_k$, $z = z^- + z^0 + z^+ \in E = E^- \oplus E^0 \oplus E^+$. Since Y_k is finite dimensional, for $\|z\| = r_k$ small enough, by (b_1) and (b_2), we get that $\Phi_\lambda(z) < 0$. ∎

LEMMA 3.2. *$a_k(\lambda) := \inf_{z\in Z_k,\, \|z\|=\rho_k} \Phi_\lambda(z) \ge 0$, where ρ_k is small enough.*

PROOF. Without loss of generality, we assume that k is large enough such that $Z_k := \oplus_{i=k}^\infty\{\mathbf{R}e_i\} \subset E^+$ and that $\lambda_k > 2\beta$. Define $\eta_k := \sup_{z\in Z_k,\, z\ne 0} |z|_{\delta_2}/\|z\|$. By (1.3), E is imbedded compactly into L^t with $t \in (2/(3-p), \infty]$ (cf. [4, Lemma 2.2]); then it is easy to check that $\eta_k \to 0$. Note that $\lambda_k|z|_2^2 \le \|z\|^2$ and that $|z|_\infty \le c\|z\|$. We have that

$$\begin{aligned}\Phi_\lambda(z) &= \frac{1}{2}\|z\|^2 - \frac{1}{2}\beta\lambda\int_{\mathbf{R}}|z|^2\,dt - \lambda\int_{\mathbf{R}} G(t,z)\,dt \\ &\ge \left(\frac{1}{2} - \frac{\beta}{\lambda_k}\right)\|z\|^2 - c\int_{\mathbf{R}}|z|^{\delta_2}\,dt \ge \left(\frac{1}{2} - \frac{\beta}{\lambda_k}\right)\|z\|^2 - c\eta_k^{\delta_2}\|z\|^{\delta_2} = c\eta_k^{\delta_2}(\rho_k)^{\delta_2},\end{aligned}$$

for $\|z\| := \rho_k := (4c\lambda_k\eta_k^{\delta_2}/(\lambda_k - 2\beta))^{1/(2-\delta_k)}$. Evidently, $\rho_k \to 0$ as $k \to \infty$. ∎

PROOF OF THEOREM 1.2. By Theorem 3.1, we observe that there exist $\lambda_n \to 1$, $z(\lambda_n) := z_n$ such that $\Phi'_{\lambda_n}|_{Y_n}(z_n) = 0$, $\Phi_{\lambda_n}(z_n) \to c_k \in [d_k(2), b_k(1)]$, as $n \to \infty$. We shall prove that $\{z_n\}$ is bounded. Otherwise, we suppose, up to a subsequence, that $\|z_n\| \to \infty$. Assume that $z_n/\|z_n\| \rightharpoonup w$, $z_n^\pm/\|z_n\| \rightharpoonup w^\pm$, $z_n^0/\|z_n\| \rightharpoonup w^0$. Since $\langle z_n^+, \phi_n\rangle - \lambda_n\langle z_n^-, \phi_n\rangle - \lambda_n\beta\int_{\mathbf{R}} z_n\cdot\phi_n\,dt - \int_{\mathbf{R}} G_z(t,z_n)\cdot\phi_n\,dt = 0$, where $\phi_n = \phi|_{Y_n}$, $\phi = \sum_{i=1}^\infty s_ie_i$. By Condition (b_2), $\langle w^+, \phi\rangle - \langle w^-, \phi\rangle - \beta\int_{\mathbf{R}} w\cdot\phi\,dt = 0$. If $w \ne 0$, we get a contradiction since $\beta \notin \sigma(-\frac{d^2}{dt^2} + L(t))$. If $w = 0$, since

$\Phi'_{\lambda_n}|_{Y_n}(z_n) = 0$, we see that $1 = \|z_n\|^{-2}(\int_{\mathbf{R}} H_z(t, z_n) \cdot (\lambda_n z_n^+ - z_n^- - z_n^0)\, dt + \|z_n^0\|^2)$. But Condition ($b_2$) and $w = 0$ imply that the right-hand side goes to zero, we arrive at a contradiction. Therefore, $\{z_n\}$ is bounded. By standard argument, it yields a critical point z^k of Φ such that $\Phi(z^k) \in [d_k(2), b_k(1)]$. Since $d_k(2) \to 0^-$, we obtain infinitely many critical points. ∎

REFERENCES

1. P.C. Carrião and O.H. Miyagaki, Existence of homoclinic solutions for a class of time-dependent Hamiltonian systems, *J. Math. Anal. Appl.* **230**, 157–172, (1999).
2. V. Coti Zelati and P.H. Rabinowitz, Homoclinic orbits for second order Hamiltonian systems possessing superquadratic potentials, *J. A. M. S.* **4**, 693–727, (1991).
3. C.-N. Chen and S.-Y. Tzeng, Existence and multiplicity results for homoclinic orbits of Hamiltonian systems, *E. J. Diff. Eqns.* **1997**, 1–19, (1997).
4. Y. Ding, Existence and multiplicity results for homoclinic solutions to a class of Hamiltonian systems, *Nonl. Anal. TMA* **25**, 1095–1113, (1995).
5. W. Omana and M. Willem, Homoclinic orbits for a class of Hamiltonian systems, *Diff. Int. Eqns.* **5**, 1115–1120, (1992).
6. P.H. Rabinowitz, Homoclinic orbits for a class of Hamiltonian systems, *Proc. R. Soc. Edinb.* **114A**, 33–38, (1990).
7. P.H. Rabinowitz, Multibump solutions of differential equations: An overview, *Chinese J. Math.* **24**, 1–36, (1996).
8. E. Serra, M. Tarallo and S. Terracini, On the existence of homoclinic solutions for almost periodic second order systems, *Ann. I. H. P. Anal. Non Linéaire* **13**, 783–812, (1996).
9. W. Zou, Variant fountain theorems and their applications, *Manuscripta Mathematica* **104**, 343–358, (2001).
10. A. Szulkin and W. Zou, Homoclinic orbit for asymptotically linear Hamiltonian systems, *J. Function Analysis* **187**, 25–41, (2001).